

நல திரட்டை/புதிய பாடத்திட்டம்/New Syllabus

## NEW

|                    |   |
|--------------------|---|
| கணக்கு             | I |
| கணிதம்             | I |
| <b>Mathematics</b> | I |

07 E I

ஆடை கூறுகீ  
மூன்று மணித்தியாலம்  
*Three hours*

|                                |                   |
|--------------------------------|-------------------|
| அம்தர கியலீம் காலை             | - தீவித்து 10 சி  |
| மேலதிக வாசிப்பு நேரம்          | - 10 நிமிடங்கள்   |
| <b>Additional Reading Time</b> | <b>10 minutes</b> |

**Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.**

### Index Number

### Instructions:

- \* *This question paper consists of two parts; Part A (Questions 1–10) and Part B (Questions 11–17).*
- \* **Part A:**  
*Answer all questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
- \* **Part B:**  
*Answer five questions only. Write your answers on the sheets provided.*
- \* *At the end of the time allotted, tie the answer scripts of the two parts together so that Part A is on top of Part B and hand them over to the supervisor.*
- \* *You are permitted to remove only Part B of the question paper from the Examination Hall.*

**For Examiners' Use only**

| (07) Mathematics I |              |       |
|--------------------|--------------|-------|
| Part               | Question No. | Marks |
| A                  | 1            |       |
|                    | 2            |       |
|                    | 3            |       |
|                    | 4            |       |
|                    | 5            |       |
|                    | 6            |       |
|                    | 7            |       |
|                    | 8            |       |
|                    | 9            |       |
|                    | 10           |       |
| B                  | 11           |       |
|                    | 12           |       |
|                    | 13           |       |
|                    | 14           |       |
|                    | 15           |       |
|                    | 16           |       |
|                    | 17           |       |
|                    | Total        |       |

| Total      |  |
|------------|--|
| In Numbers |  |
| In Words   |  |

| Code Numbers     |   |
|------------------|---|
| Marking Examiner |   |
| Checked by:      | 1 |
|                  | 2 |
| Supervised by:   |   |

## Part A

1. Let  $A = \{x \in \mathbb{R} : |x + 1| \leq 2\}$  and  $B = \{x \in \mathbb{R} : |x - 1| > 1\}$ . Find  $A \cap B$ ,  $A \cup B$  and  $A \cap B'$ .

2. Let  $A$  and  $B$  be subsets of a universal set  $S$ . Show that  $(A \cup B) \cap (A \cap B)' = (A \setminus B) \cup (B \setminus A)$ .

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3. Show that the compound proposition  $(p \wedge q) \vee r$  and the compound proposition  $(p \Rightarrow \neg q) \Rightarrow r$  are logically equivalent.

4. Using the method of contradiction, prove that if  $n^2 + 6n + 3$  is even, then  $n$  is odd.

5. Solve the equation  $\log_3 x = 2 - \log_3(6 - x)$  for  $x$ .

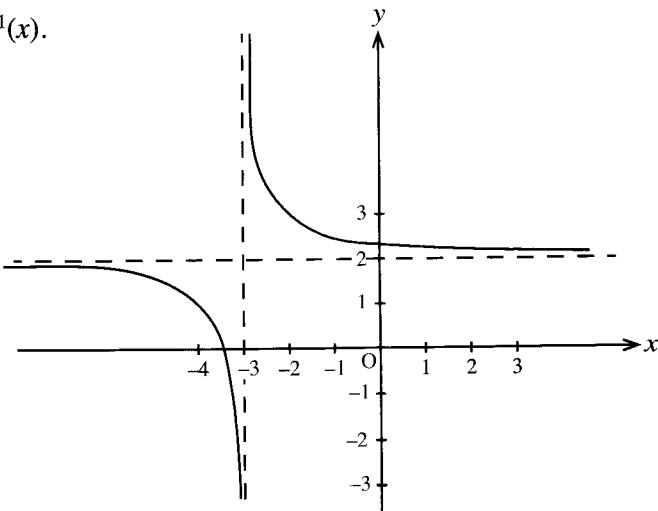
When  $x$  takes this value, solve the equation  $x^y = 2 - x^{-y}$  for  $y$ . (Hint: Use the substitution  $u = x^y$ .)

6. Find all real values of  $x$  satisfying the inequality  $x + \frac{6}{x+1} > 4$ .

7. The graph of  $f(x) = \frac{1}{x+a} + b$  is shown in the diagram. Using the information given there, write down the values of the constants  $a$  and  $b$ , and find  $f^{-1}(x)$ .

Given that  $g(x) = x - 5$ , solve  $f^{-1}(g(x)) = 4$ .

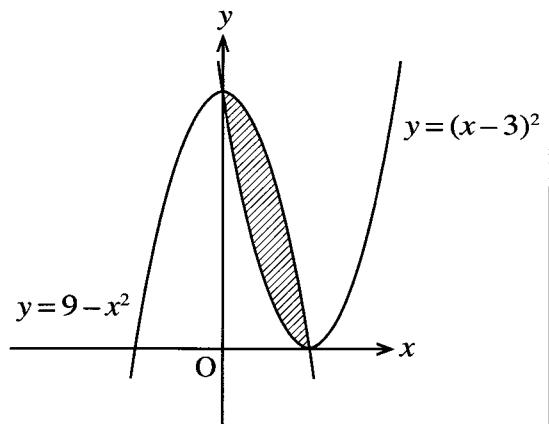
.....



8. Write down the equation of the straight line  $l$  passing through the point  $A \equiv (0, 3)$  with gradient  $-2$ . The line  $l$  meets the line  $y = mx$  at the point  $B$ , where  $m$  ( $\neq -2$ ) is a constant. Find the  $x$ -coordinate of  $B$  in terms of  $m$ .

Given that the area of the triangle  $OAB$  is  $\frac{9}{2}$  square units, where  $O$  is the origin, find the possible values of  $m$ .

9. The radius  $r$  m of a right circular cylinder is increasing at a rate of  $0.5 \text{ m s}^{-1}$  and its height  $h$  m is decreasing at a rate of  $0.2 \text{ m s}^{-1}$ . Show that the rate of change of the volume  $V \text{ m}^3$  of the cylinder is given by  $\frac{dV}{dt} = \frac{\pi r}{5}(5h - r)$ .



## நல திரட்டை/புதிய பாடத்திட்டம்/New Syllabus

அதிகார போடு கல்வி பறு (உயர் பேல்) விழாக்கு, 2020  
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரிசீலனை, 2020  
General Certificate of Education (Adv. Level) Examination, 2020

# கணக்கு கணிதம் **Mathematics**

07 E I

## Part B

\* Answer five questions only.

11. (a) A survey was carried out using 100 students in a class to find out which branches of mathematics they liked from amongst Algebra and Geometry. It was found that the number of students who liked Geometry was 10 more than twice the number of students who liked Algebra. It was also found that 80 students liked only one branch and 10 students did not like both.

Find the number of students who liked

- (i) Algebra
- (ii) Geometry
- (iii) both Algebra and Geometry.

(b) Using truth tables, determine whether each of the following compound propositions is a tautology or a contradiction.

- $(p \wedge q) \wedge (q \Rightarrow \sim p)$
- $(p \wedge q \wedge r) \vee (p \wedge q \wedge (\sim r)) \vee (\sim(p \wedge q))$

12.(a) Using the Principle of Mathematical Induction, prove that

$$\sum_{r=1}^n r(3r+2) = \frac{n}{2}(n+1)(2n+3) \quad \text{for all } n \in \mathbb{Z}^+.$$

(b) Let  $U_r = \frac{r^2 + r - 1}{(r+1)^2(r+2)^2}$  for  $r \in \mathbb{Z}^+$ .

Verify that  $U_r = \frac{r}{(r+1)^2} - \frac{(r+1)}{(r+2)^2}$  for  $r \in \mathbb{Z}^+$ .

Show that  $\sum_{r=1}^n U_r = \frac{1}{4} - \frac{(n+1)}{(n+2)^2}$  for  $n \in \mathbb{Z}^+$ .

**Hence, show that**  $\sum_{r=1}^{\infty} U_r$  **is convergent and find its sum.**

**Deduce** that  $\sum_{r=20}^{\infty} U_r = \frac{20}{441}$ .

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13.(a) Let  $k$  ( $\neq 0$ ) be a real constant. It is given that the quadratic equation  $2kx^2 + 12x + 2k - 5 = 0$  has real roots. Show that  $2k^2 - 5k - 18 \leq 0$ .

Find the maximum and the minimum of possible values of  $k$ .

Let  $\alpha$  and  $\beta$  be the roots of the equation  $2kx^2 + 12x + 2k - 5 = 0$ .

Find the quadratic equation whose roots are  $2(\alpha + \beta)$  and  $3\alpha\beta$ .

(b) Let  $f(x) = x^3 + px^2 + q$  and  $g(x) = x^3 + qx^2 - p$ , where  $p$  and  $q$  are real numbers. It is given that  $(x+2)$  is a factor of  $f(x)$  and that when  $g(x)$  is divided by  $(x+1)$ , the remainder is  $-8$ .

Find the values of  $p$  and  $q$ .

For these values of  $p$  and  $q$ , find the least value of  $f(x) - g(x)$ .

14.(a) Let  $a, b \in \mathbb{R}$ . The expansion of  $(1+ax)^8$ , in ascending powers of  $x$ , discarding the terms involving powers of  $x$  greater than two is  $1 + 24x + bx^2$ . Show that  $a = 3$  and  $b = 252$ .

Hence, find an approximate value for  $(1.03)^8 + (0.97)^8$ .

(b) A person wants to take a loan of Rs. 2000000 from a bank, to be paid back in 10 years. The bank charges an annual interest of 6% compounded monthly. Let Rs.  $A_n$  be the outstanding amount after paying the  $n^{\text{th}}$  installment at the end of the  $n^{\text{th}}$  month, where  $n \leq 120$ .

Show that  $A_1 = 1.005A - x$ , where  $A$  is the loan amount and  $x$  is the monthly installment.

Obtain expressions for  $A_2$  and  $A_3$ , and write down  $A_n$  in terms of  $A$ ,  $x$  and  $n$ .

Hence, find the value of  $x$ .

15. Let  $A \equiv (1, 1)$  and  $B \equiv (5, 9)$ .

Find the equation of the straight line  $AB$  and show that the point  $C \equiv (4, 2)$  does not lie on the line  $AB$ .

The line perpendicular to  $AB$  and passing through  $C$ , intersects  $AB$  at the point  $D$ .

Find the coordinates of  $D$  and show that  $AD:DB = 1:3$ .

Also, find the coordinates of the point  $E$  such that  $ADCE$  is a rectangle.

Let  $F$  be the point of intersection of the line  $AB$  and the line  $x + y = k$ . The line passing through the point  $F$  and parallel to the line  $AC$  passes through the point  $E$ . Find the value of the constant  $k$ .

16.(a) Evaluate  $\lim_{x \rightarrow 2} \frac{x^4 - 16}{\sqrt{x} - \sqrt{2}}$ .

(b) Differentiate each of the following with respect to  $x$ :

$$(i) (2 + 3x)^5 (1 + x^2)^{10} \quad (ii) \frac{\ln x}{3 \ln x + 1} \quad (iii) \sqrt{x} e^{-(x^2-1)}$$

(c) A closed rectangular box needs to be constructed such that the length of the base is 3 times its width. It costs 100 rupees per square meter for the top and the bottom faces, and 60 rupees per square meter for the sides of the box. If the volume of the box must be  $60 \text{ m}^3$ , show that the cost  $C$  (in rupees) to make the box is given by  $C = 600x^2 + \frac{9600}{x}$ , where  $x \text{ m}$  is the width of the base of the box.

Determine the value of  $x$  that minimizes the cost to make the box.

17.(a) Using the method of **integration by parts**, find  $\int x^3 (\ln x)^2 dx$ .

(b) The following table gives the values of the function  $f(x) = \ln(1 + x^2)$ , correct to three decimal places, for values of  $x$  between 1 and 2.5 at intervals of length 0.25.

|        |       |       |       |       |       |       |       |
|--------|-------|-------|-------|-------|-------|-------|-------|
| $x$    | 1.00  | 1.25  | 1.50  | 1.75  | 2.00  | 2.25  | 2.50  |
| $f(x)$ | 0.693 | 0.941 | 1.179 | 1.402 | 1.609 | 1.802 | 1.981 |

Using **Simpson's rule**, find an approximate value for  $I = \int_1^{2.5} \ln(1 + x^2) dx$ .

Hence, find an approximate value for  $\int_1^{2.5} \ln(e^{2x} \sqrt{1+x^2}) dx$ .

\* \* \*

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நல திரட்டை/புதிய பாடத்திட்டம்/New Syllabus

අධ්‍යාපන පොදු සහතික පත්‍ර (උස්ස පෙළ) විභාගය, 2020  
කළුවිප පොතුන් තාරාතරප පත්තිර (ඉයුර තරප) පරිශ්‍යා, 2020  
General Certificate of Education (Adv. Level) Examination, 2020

|                    |           |
|--------------------|-----------|
| கணக்கு             | II        |
| கணிதம்             | II        |
| <b>Mathematics</b> | <b>II</b> |

07 E II

பூரை ஒன்றை  
மூன்று மணித்தியாலம்  
*Three hours*

|                                |                     |
|--------------------------------|---------------------|
| அமுலர் கியெலி காலை             | - தீவின்து 10 சி    |
| மேலதிக வாசிப்பு நேரம்          | - 10 நிமிடங்கள்     |
| <b>Additional Reading Time</b> | <b>- 10 minutes</b> |

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## Index Number

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**Part B:**  
Answer **five** questions only. Write your answers on the sheets provided.
- \* At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.
- \* You are permitted to remove **only Part B** of the question paper from the Examination Hall.
- \* Statistical tables will be provided.

**For Examiners' Use only**

| (07) Mathematics II |              |       |
|---------------------|--------------|-------|
| Part                | Question No. | Marks |
| A                   | 1            |       |
|                     | 2            |       |
|                     | 3            |       |
|                     | 4            |       |
|                     | 5            |       |
|                     | 6            |       |
|                     | 7            |       |
|                     | 8            |       |
|                     | 9            |       |
|                     | 10           |       |
| B                   | 11           |       |
|                     | 12           |       |
|                     | 13           |       |
|                     | 14           |       |
|                     | 15           |       |
|                     | 16           |       |
|                     | 17           |       |
|                     | Total        |       |

| <b>Total</b> |  |
|--------------|--|
| In Numbers   |  |
| In Words     |  |

## Code Numbers

|                  |   |
|------------------|---|
| Marking Examiner |   |
| Checked by:      | 1 |
|                  | 2 |
| Supervised by:   |   |

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## Part A

1. Let  $a, b, c \in \mathbb{R}$ .

$$\text{Show that } \begin{vmatrix} a & a & 2a+b+c \\ b & a+2b+c & b \\ a+b+2c & c & c \end{vmatrix} = -2(a+b+c)^3.$$

2. Let  $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 3 & 2 \\ -1 & 2 \\ 1 & 0 \end{pmatrix}$  and  $C = \begin{pmatrix} -1 & 0 \\ 1 & 3 \end{pmatrix}$ . Find  $AB$  and  $BC$ .

Verify that  $A(BC) = (AB)C$ .

3. The mean and the standard deviation of a set of 10 observations are 5 and 10, respectively. Find the sum and the sum of squares of these observations. If another observation of value 5 is added to this set, find the new values of the mean and the standard deviation.

4. The mean, median and the standard deviation of a distribution are 28, 32 and 5, respectively. Calculate Karl Pearson's coefficient of skewness and describe the shape of the distribution. Is the mean a fair measurement of the central tendency for this distribution? Give reasons for your answer.

5. The speed of vehicles travelling on a certain section of a highway, is normally distributed with mean  $90 \text{ km h}^{-1}$  and standard deviation  $10 \text{ km h}^{-1}$ . Find the probability that the speed of a randomly selected vehicle is between  $85 \text{ km h}^{-1}$  and  $100 \text{ km h}^{-1}$ .

6. It is found from previous records that 10% of the bolts produced by a machine are defective. If 5 bolts produced by this machine are chosen at random, find the probability that  
(i) exactly 3 bolts are defective,  
(ii) more than 2 bolts are **non-defective**.

7. In a group consisting of 30 cricketers, 20 have played for club A and 15 have played for club B. Every cricketer has played for at least one of these clubs. Find the probability that a cricketer selected at random has played for club B, given that he has played for club A.

8. Let  $A$  and  $B$  be two events of a sample space  $S$  such that  $P(A) = \frac{3}{8}$ ,  $P(A \cap B) = \frac{1}{8}$  and  $P(A \cup B) = \frac{3}{4}$ . Find

(i)  $P(B)$ , (ii)  $P(A' \cap B)$  and (iii)  $P(A'|B)$ .

9. The probability mass function of a discrete random variable  $X$  is given below:

|            |     |      |     |      |     |
|------------|-----|------|-----|------|-----|
| $x$        | 1   | 2    | 3   | 4    | 5   |
| $P(X = x)$ | $p$ | $2p$ | $p$ | $2p$ | $p$ |

Find the value of the constant  $p$  and show that  $E(X) = 3$ .

Let  $Y$  be the random variable given by  $3X - 4$ . Find  $P(Y > X)$ .

10. A continuous random variable  $X$  has probability density function  $f(x)$  given by

$$f(x) = \begin{cases} kx - x^2 & , \text{ if } 0 \leq x \leq 1, \\ 0 & , \text{ otherwise,} \end{cases}$$

where  $k$  is a constant.

Show that  $k = \frac{8}{3}$  and find  $E(X)$ .

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## நல கிரட்டையை/புதிய பாடத்திட்டம்/New Syllabus

අධ්‍යාපන පොදු සහතික පත්‍ර (උස්ස පෙළ) විභාගය, 2020  
කළුවිප පොත්‍හත තරාතරුප පත්තිර (ඉයර තරුප පරිශාස, 2020  
General Certificate of Education (Adv. Level) Examination, 2020

கணிதம்  
Mathematics II

07 E II

## Part B

\* Answer **five** questions only.

11. A factory manufactures tables and chairs. The production of each item requires three operations: cutting, assembling and finishing.

For cutting, assembling and finishing, the maximum number of hours that can be used are 600, 160 and 280, respectively. The following table gives the number of hours required for each operation in producing each item and the profit per item sold.

|       | Number of hours for cutting | Number of hours for assembling | Number of hours for finishing | Profit (in thousands of rupees) |
|-------|-----------------------------|--------------------------------|-------------------------------|---------------------------------|
| Table | 5                           | 1                              | 1                             | 12                              |
| Chair | 6                           | 2                              | 4                             | 15                              |

The factory wishes to maximize the profit.

- (i) Formulate this as a linear programming problem.
- (ii) Sketch the feasible region.
- (iii) Using the graphical method, find the solution of the problem formulated in part (i) above.
- (iv) Due to shortage of storage space, the factory has to limit the total number of tables and chairs produced to at most 108. Find the decrease in the profit due to above limitation, if the factory still wishes to maximize the profit.

12.(a) Let  $A = \begin{pmatrix} 4 & 7 \\ -1 & -2 \end{pmatrix}$ . Write down  $A^{-1}$ .

Let  $\mathbf{B} = \begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix}$ .

Find the matrix  $C$  such that  $AC = B$  and show that

$$\mathbf{AC} - \mathbf{CA} = \begin{pmatrix} 20 & 43 \\ -11 & -20 \end{pmatrix}.$$

Find the matrix  $D$  such that  $AC - DA = O$ , where  $O$  is the zero matrix of order 2.

(b) Let  $a \in \mathbb{R}$ . Write the pair of **simultaneous** equations

$$(a - 5)x + 3y = a$$

$$-4x + (a + 2)y = 1$$

in the form  $\mathbf{P}\mathbf{X} = \mathbf{Q}$ , where  $\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}$ , and  $\mathbf{P}$  and  $\mathbf{Q}$  are matrices to be determined.

Express  $\Delta = \begin{vmatrix} (a-5) & 3 \\ -4 & (a+2) \end{vmatrix}$  as a quadratic function of  $a$ .

Show that the roots of the equation  $\Delta = 0$  are  $a = 1$  and  $a = 2$ .

Show that the above pair of equations has

- (i) infinitely many solutions when  $a = 1$ ,
- (ii) no solution when  $a = 2$ ,
- (iii) a unique solution when  $a = 3$ .

13.(a) An unbiased cubic die with faces marked 1, 2, 2, 3, 3, 4 is tossed twice. Let  $A$  be the event that the sum of the numbers obtained is 4 and  $B$  be the event that the sum of the numbers obtained is even.

Find  $P(A)$ ,  $P(B)$  and  $P(A|B)$ .

(b) Four digits from the set of digits {1, 2, 3, 4, 5, 6} are chosen without replacement and a 4-digit number is made.

- (i) How many different 4-digit numbers can be made?
- (ii) How many of these 4-digit numbers start with 3 or 5?

(c) A team of four people must be selected from a group of four males and two females.

- (i) How many different teams of four people can be selected?
- (ii) Find the probability that both females are selected to these teams.

14. A box  $X$  contains 4 red cards and 6 blue cards. A box  $Y$  contains 3 red cards and 2 blue cards. A biased coin with  $\frac{2}{3}$  as the probability of getting a head is tossed. If the outcome is a head, 2 cards are drawn from the box  $X$ , at random without replacement, and if it is a tail, 2 cards are drawn from the box  $Y$ , at random without replacement. Find the probability that

- (i) both cards drawn are red,
- (ii) at least one of the cards drawn is red,
- (iii) the two cards drawn are of different colours,
- (iv) the two cards drawn are of different colours, given that at least one of the cards drawn is red.

15.(a) The time  $X$ , measured in minutes, between consecutive arrivals of buses to a certain bus stop is exponentially distributed with probability density function

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where  $\lambda (> 0)$  is a parameter.

If the mean number of buses that arrive at the bus stop in an hour is 12, find the value of  $\lambda$ .

(i) After a bus arrives at the bus stop, find the probability that the time taken for the next bus to arrive at the bus stop is

- between one minute to three minutes,
- less than five minutes.

(ii) If it is given that five minutes has already passed from the arrival of a bus to the bus stop, find the probability that it takes at least an additional two minutes for the next bus to arrive.

(b) A continuous random variable  $X$  is uniformly distributed over the interval  $[a, b]$ .  
Find the values of  $a$  and  $b$  such that  $P(X < 16) = 0.4$  and  $P(X > 21) = 0.2$ .

16. Hundred students faced an entrance test. The frequency distribution of the marks they obtained is given in the following table:

| Marks    | frequency |
|----------|-----------|
| 0 – 20   | 15        |
| 20 – 40  | 20        |
| 40 – 60  | 40        |
| 60 – 80  | 15        |
| 80 – 100 | 10        |

(i) Estimate each of the following:

- the mean,
- the standard deviation,
- the median,
- the inter quartile range and
- the mode

of the marks.

(ii) After rescrutiny, it was discovered that the marks of two answer scripts should be changed as follows:

| Marks before rescrutiny | Marks after rescrutiny |
|-------------------------|------------------------|
| 50                      | 62                     |
| 70                      | 75                     |

Find the mean of the new distribution of marks.

17. The duration of activities of a project and the flow of activities are given in the following table:

| Activity | Preceding Activity (Activities) | Duration (in Weeks) |
|----------|---------------------------------|---------------------|
| A        | —                               | 03                  |
| B        | A                               | 08                  |
| C        | A                               | 05                  |
| D        | A                               | 03                  |
| E        | B                               | 06                  |
| F        | C                               | 03                  |
| G        | E, F                            | 04                  |
| H        | D, F                            | 06                  |
| I        | G, H                            | 03                  |

- (i) Construct the project network.
- (ii) Prepare an activity schedule that includes earliest start time, earliest finish time, latest start time, latest finish time and float for each activity.
- (iii) Find the total duration of the project.
- (iv) What are the activities that can be delayed without extending the total duration of the project?
- (v) Write down the critical path of this project.
- (vi) Suppose that the duration of the activity D has to be extended by two weeks due to an unexpected matter. Determine whether the project could still be completed within the total duration calculated in part (iii) above.

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