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கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2016 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2016

ගණිතය I கணிதம் I **Mathematics I**	**07 E I**	පැය තුනයි மூன்று மணித்தியாலம் ***Three hours***

Index Number							

Instructions:

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  Part A (Questions 1 - 10) and **Part B** (Questions 11 - 17).
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  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(07) Mathematics I		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
	Percentage	

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Code Numbers

Marking Examiner 1	
Marking Examiner 2	
Checked by:	
Supervised by:	

[see page two

Part - A

1. Let A, B and C be subsets of a universal set S. Show that $(A \cap B) \cup (A' \cap B) = B$.
   Deduce that if $A \cap B = A \cap C$ and $A' \cap B = A' \cap C$, then $B = C$.

2. Let $A = \{x \in \mathbb{R} : |x-1| \geq 1\}$, $B = \{x \in \mathbb{R} : |x| < 2\}$ and $C = \{x \in \mathbb{R} : x \leq 1\}$.
   Find $A \cap B$, $A \cap C$ and $B \cup C$, and verify that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

[see page three

3. Let $f(x) = e^{2x^2}$ for $x \in \mathbb{R}$. A relation R on $\mathbb{R}$ is defined by aRb if $f(a) = f(b)$. Prove that the relation R is an equivalence relation on $\mathbb{R}$ and find the equivalence class of 1.

4. Let $f(x) = \frac{2x}{3x-1}$ for $x \neq \frac{1}{3}$. Show that f is one-to-one.

Find $f^{-1}(x)$ and show further that $f^{-1}(2f(1)) = \frac{1}{2}$.

[see page four

5. Solve the simultaneous equations $2^{x+3} + 3y - 10 = 0$ and $x + \log_2 y = 0$ for x and y.

6. Let $f(x) = \begin{vmatrix} -2-x & -3 & -1 \\ 1 & 2-x & 1 \\ 3 & 3 & 2-x \end{vmatrix}$.

Without expanding the determinant, show that $(x + 1)$ is a factor of $f(x)$.

Hence or otherwise, find the roots of the equation $f(x) = 0$.

[see page five

7. The points P and Q have the coordinates $(-4, 4)$ and $(2, 6)$ respectively. The straight line PQ is perpendicular to the straight line l that passes through the point Q. Find the equation of l.

If the straight line l meets the y-axis at the point R, show that $PQ = QR$.

8. A circle has a diameter with end-points at $(-7, 4)$ and $(1, -2)$. Find the coordinates of the centre and the equation of the circle.

Show that the straight line $3x + 4y = 10$ intersects this circle.

[see page six

9. The radius and the height of a right circular cylinder are increasing at rates 7 $m\,s^{-1}$ and 3 $m\,s^{-1}$ respectively. Find the rate at which the volume of the cylinder is changing when the radius and the height of the cylinder are 6 m and 5 m respectively.

10. The gradient of the tangent drawn to the curve $y = ax^2 + bx$ at the point $P \equiv (1, 2)$ on it is 3, where a and b are constants. Find the values of a and b.

The normal drawn to the curve at P meets the curve again at Q. Find the x-coordinate of Q.

[see page seven

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கல்விப் பொதுத் தராதரப் பத்திர (உயர் தரப்) பரீட்சை, 2016 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2016

ගණිතය I
கணிதம் I
Mathematics I

07 E I

Part B

* Answer **five** questions only.

11. (*a*) Let $p \in \mathbb{R}$. Show that the roots of the equation $p^2(x-1)^2 + (x-2)^2 = 2$ are real and distinct. Let α and β be the roots of the above equation. Show that $\alpha + \beta = 2\alpha\beta$.

(*b*) Let, $f(x) = ax^4 + x^3 - x^2 - x - b$, where a and b are real constants. It is given that $(x-1)$ is a factor of $f(x)$ and that the remainder, when $f(x)$ is divided by $(x-2)$ is 33. Find the values of a and b.

Show that $(x+1)$ is also a factor of $f(x)$.

Express $f(x)$ as a product of two linear factors and a quadratic factor which is positive for all $x \in \mathbb{R}$.

12. (*a*) Using the **principle of Mathematical Induction**, prove that $\sum_{r=1}^{n} r(2r+1) = \frac{n}{6}(n+1)(4n+5)$ for all $n \in \mathbb{Z}^+$

(*b*) Let $U_r = \frac{1}{(3r-8)(3r-2)}$ and let $f(r) = \lambda \frac{(3r+2)}{(3r-8)}$ for $r \in \mathbb{Z}^+$, where $\lambda \in \mathbb{R}$.

Find the value of λ such that $U_r = f(r) - f(r+2)$ for $r \in \mathbb{Z}^+$.

Hence, find $\sum_{r=1}^{n} U_r$.

Show that $\sum_{r=1}^{\infty} U_r$ is convergent.

Let $V_r = 3U_r + 2$ for $r \in \mathbb{Z}^+$. Find $\sum_{r=1}^{n} V_r$.

Is $\sum_{r=1}^{\infty} V_r$ convergent? Justify your answer.

13. (*a*) From a group of 8 men and 5 women, 6 people are to be selected to serve on a committee. Find the number of different ways the committee can be formed if it must include

(i) exactly 3 men and 3 women,

(ii) at most 3 women,

(iii) at least 3 women.

(*b*) How many different 7-digit numbers can be formed using the digits 1, 2, 2, 2, 4, 4 and 5?

[see page eight

14. (*a*) Let $\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -1 & 1 & 2 \end{pmatrix}$.

Find **AB** and **BA**.

Verify that $(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$ and $(\mathbf{BA})^T = \mathbf{A}^T\mathbf{B}^T$; where $\mathbf{P}^T$ denotes the transpose of a matrix **P**.

(*b*) Let $\mathbf{C} = \begin{pmatrix} -1 & 0 & 0 \\ 1 & 2 & 1 \\ -2 & -3 & -2 \end{pmatrix}$.

Find $\mathbf{C}^2 - \mathbf{I}$ and show that $\mathbf{C}(\mathbf{C}^2 - \mathbf{I}) = \mathbf{I} - \mathbf{C}^2$, where **I** is the unit matrix of order 3.

Hence, find $\mathbf{C}^{-1}$.

Also, find the 3×3 matrix **D** such that $\mathbf{CD} = \mathbf{I} + 2\mathbf{C}$.

15. (*a*) Find the constant term in the binomial expansion of $\left(x - \frac{2}{x^2}\right)^9$.

(*b*) Show that $\left(\sqrt{2}+1\right)^6 + \left(\sqrt{2}-1\right)^6 = 198$.

(*c*) A person opened a savings account, which pays 10% compound annual interest, on 1st January 2000 by depositing 100 000 rupees. He deposited 10 000 rupees on 1st January of each of the following five years. Assuming that he made no other deposits or withdrawals, find the balance in the account on 1st January 2010.

16. Show that the straight line $y = mx + c$ is tangent to the circle $(x-p)^2 + (y-q)^2 = r^2$ if and only if $r^2(m^2+1) = (q - mp - c)^2$.

Let $k \in \mathbb{R}$. It is given that the straight line $x + y = k$ is tangent to the circle $x^2 + y^2 - 4x - 2y - 13 = 0$.

Find the **two** values of k.

For each of these values of k, find the coordinates of the point of contact.

Find the equation of the circle passing through these two points of contact and the origin.

17. (*a*) Evaluate $\lim_{x \to 1} \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$.

(*b*) Differentiate each of the following functions with respect to x:

(i) $\sqrt{\frac{2x+1}{2x-1}}$ (ii) $xe^{2x} + e^{-x^2}$ (iii) $\ln(x^2+1)$

(*c*) A window has the shape of a rectangle surmounted by an equilateral triangle. The total perimeter of the window is 6 m. Find the maximum area of the window.

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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2016 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2016 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2016

ගණිතය II கணிதம் II **Mathematics II**	**07 E II**		පැය තුනයි மூன்று மணித்தியாலம் ***Three hours***

Index Number							

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* *Statistical tables will be provided.*

For Examiners' Use only

(07) Mathematics II

Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
	Percentage	

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Code Numbers

Marking Examiner 1	
Marking Examiner 2	
Checked by:	
Supervised by:	

[see page two

Part - A

1. Find all real values of x satisfying the inequality $\frac{x+2}{5-2x} \geq 4$.

2. Shade the region in the xy-plane satisfying all three inequalities $3y - x \geq 0$, $y + x - 4 \leq 0$ and $y - x - 4 \leq 0$.

[see page three

5. Express $\frac{1}{x^2(x-1)}$ in partial fraction. **Hence,** find $\int \frac{1}{x^2(x-1)}\,dx$.

6. The probability distribution of a discrete random variable X is given below:

x	0	1	2
$P(X=x)$	$\frac{1}{10}$	$\frac{1}{2}$	$\frac{2}{5}$

Find $E(X)$ and $Var(X)$.

[see page five

3. Express $\frac{1}{2}\sin\left(x - \frac{\pi}{6}\right) + \cos x$ in the form $R\sin(x + \alpha)$, where $R(> 0)$ and $\alpha\left(0 < \alpha < \frac{\pi}{2}\right)$ are real constants.

Hence, solve the equation $\frac{1}{2}\sin\left(x - \frac{\pi}{6}\right) + \cos x = \frac{3}{4}$.

4. Using **integration by parts**, evaluate $\int_1^2 x(\ln x)^2\, dx$.

[see page four

7. On a certain day, the probability that two workers take leave is $\frac{1}{8}$ and the probability that only one of them takes leave is $\frac{1}{2}$. Assume that they take leave independently. Find the probability that **none** of the two workers take leave on that day.

8. Let A and B be two events defined on a sample space S. In the usual notation, $P(A) = 0.8$, $P(B) = 0.3$ and $P(A \cup B) = 0.9$. Find $P(A' \cap B')$ and $P(A \cap B')$, where A' and B' denote the complements of A and B respectively.

[see page six

9. A continuous random variable X has probability density function

$$f(x) = \begin{cases} k(x-2)^2 & , \text{ if } 0 \leq x \leq 4, \\ 0 & , \text{ other wise.} \end{cases}$$

Find the value of the constant k and the value of the first quartile.

10. In a computer game, if the player wins a game, the probability that he will win the next game is 0.9. If the player loses a game, the probability that he will win the next game is 0.4. Consider 'win' and 'lose' as the states of a two-state Markov chain.

(i) Write down the one-step transition probability matrix.

(ii) Player consecutively plays three games. If he wins the first game, find the probability that the player wins the third game.

Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2016 අගෝස්තු
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General Certificate of Education (Adv. Level) Examination, August 2016

ගණිතය II
கணிதம் II
Mathematics II

07 E II

Part B

* Answer **five** questions only.

11. A farmer has 20 acres of land for growing leeks and okra. The farmer needs to decide how much of each crop to grow. The cost per acre for leeks is Rs. 30 000 and for okra it is Rs. 20 000. The farmer has allocated to spend Rs. 480 000 for this purpose. In order to cultivate the crops, leeks requires 1 man-day per acre and okra requires 2 man-days per acre. There are 36 man-days available for this purpose. The profit on leeks is Rs. 100 000 per acre and for okra it is Rs. 120 000 per acre.

(i) Formulate this as a linear programming problem.

(ii) Sketch the feasible region.

(iii) Find the number of acres of each crop the farmer should cultivate to maximize profit.

How does the optimal solution change, if the farmer had 38 man-days available for this purpose?

12. (*a*) Find the coordinates of the points of intersection of the curves $y = -3\cos^2 x$ and $y = 3\sin^2 x + 4\cos x - 5$ in the range $0 \le x \le \frac{\pi}{2}$.

(*b*) Solve $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$.

(*c*) For a triangle ABC, in the usual notation state the Sine Rule.

Hence, show that $b \sec A = c\left[1 + \tan A \cot C\right]$.

13. (*a*) Using a suitable substitution, find $\int x\left(1 + x^2\right)^{\frac{1}{3}} \mathrm{d}x$.

(*b*) The following table gives the values of the function $f(x) = \dfrac{x+1}{x^2 + 2x - 2}$, correct to three decimal places, for values of x between 1 and 2 at intervals of length 0.2.

x	1	1.2	1.4	1.6	1.8	2
$f(x)$	2.000	1.1957	0.8696	0.6915	0.5785	0.500

Using Simpson's Rule, find an approximate value for $I = \int_1^2 \dfrac{x+1}{x^2 + 2x - 2}\,\mathrm{d}x$ correct to two decimal places.

Hence, find an approximate value for $\ln 2$.

[*see page eight*

14. (*a*) Twenty measurements with a mean of 30 units and a standard deviation of 4 units have been obtained from a smoke detection test. A linear transformation is applied to these measurements so that the mean is increased by 20 units and the standard deviation is increased by 2 units. Find the linear transformation.

(i) If the median of the original measurements is 35 units, find the median of the transformed data.

(ii) If the range of the original measurements is 18 units, find the range of the transformed data.

(*b*) A blood test correctly indicates that a person is infected with dengue virus with probability 0.9, and incorrectly indicates that a person is infected with dengue virus with probability 0.02. If 13% of persons in a certain town were indicated by the test as infected with dengue virus, find the probability that a randomly selected person from this town is actually infected with dengue virus.

Also, find the probability that a randomly selected person from this town is actually infected with dengue virus and the test correctly indicates that the person is infected with dengue virus.

15. Records of an insurance company indicate that 25% of the claims are for accidents with owner as the driver. Furthermore, 60% of these claims are for minor repairs and the rest are for major repairs. Also, among the claims in which owner is not the driver, 80% are for minor repairs and the rest are for major repairs. A claim is randomly selected. Find the probability that the selected claim is for major repairs.

Given that the claim is for major repairs, find the probability that the owner was the driver.

Now, three claims are randomly selected. Find

(i) the expected number of claims for major repairs,

(ii) the probability that none of the claims are for major repairs,

(iii) the probability that all three claims are for major repairs, given that at least one of the claims is for major repairs.

16. The weights of a certain group of persons, measured in kilogrammes are normally distributed with mean μ and standard deviation σ. The probabilities that the weight of a randomly selected person from this group exceeds 60 and 65 kilogrammes are 0.1587 and 0.0228 respectively. Find the values of μ and σ.

(i) Find the probability that the weight of a randomly selected person from this group exceeded 50 kilogrammes,

(ii) Given that the weight of a randomly selected person form this group exceeded 50 kilogrammes, find the probability that this weight is less than 65 kilogrammes.

(iii) Two persons from this group are randomly selected. Find the probability that the weight of only one person is more than 50 kilogrammes.

[see page nine

17. Number of buses X arriving at a bus stand during a 15-minute time interval follows a Poisson distribution with probability mass function given by $P(X = x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$ for $x = 0, 1, 2, \ldots$.

The probability that only a single bus arrives during a 15-minute time interval is twice as the probability of **no** bus arrives in the interval. Find λ.

(i) Find the probability that at least one bus arrives during a 15-minute interval. (you may take $e^{-2} \approx 0.1353$.)

(ii) Suppose 2% of the buses arriving at the bus stand are overloaded. Find the expected value of the number of overloaded buses arriving at the bus stand in a 15-minute interval.

(iii) Suppose that the number of buses arriving at any time interval is independent of the number of buses arriving at any other non-overlapping interval. If a bus arrived at the bus stand just prior to 7.00 a.m., find the probability that the next bus will arrive during the time interval from 7.15 a.m. to 7.30 a.m..

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