ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2017 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2017 ஓகஸ்ட்
General Certificate of Education (Adv. Level) Examination, August 2017

ගණිතය I
கணிதம் I
Mathematics I

07 E I

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

Index Number							

Instructions:

* *This question paper consists of two parts;*

  Part A (Questions **1 – 10**) and **Part B** (Questions **11 – 17**).

* **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
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* *At the end of the time allotted, tie the answers of the two parts together so that* **Part A** *is on top of* **Part B** *before handing them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(07) Mathematics I		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
	Percentage	

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Letters	

Code Numbers

Marking Examiner 1.	
Marking Examiner 2.	
Marks Checked by	
Supervised by	

[see page two

Part A

1. Let A, B and C be subsets of a universal set S. Show that $(A \cap B') \cap (B \cap C')' = A \cap B'$.

2. Considering $U = \{n \in \mathbb{Z} : 1 \leq n \leq 10\}$ as the universal set, let $A = \{1,2,4,5,6\}$, $B = \{2,4,10\}$ and $C = \{2,9,10\}$. Find $A \cap B'$, $A \cap C'$, and $(B \cap C)'$ verify that $A \cap (B \cap C)' = (A \cap B') \cup (A \cap C')$.

[*see page three*

0013

3. Let $S = \{n \in \mathbb{Z} : 1 \leq n \leq 20\}$. A relation R on S is defined by mRn if $m-n$ is a multiple of 4. Show that the relation R is an equivalence relation on S and find the equivalence class of 2.

4. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = 4x^3 - 5$. Show that the function f is one-to-one and onto, and find $f^{-1}(x)$. Show that $f^{-1}\left(\frac{115}{2}\right)$ is a rational number.

[see page four

5. Solve the following simultaneous equations for x and y:

$$16^{y} = 2^{2(2x-18)} \quad \text{and} \quad \log_5(x+y) = 1 + \log_3(x-y).$$

6. Let $f(x) = \begin{vmatrix} x^2 & 2-x & 1 \\ 1 & x^2 & 0 \\ x-1 & 0 & -x \end{vmatrix}$.

Without expanding the determinant, show that (x^2-1) is a factor of $f(x)$. Also, find the solutions of the equation $f(x) = x(x^2-2)$.

[see page five

7. Let $A \equiv (1, 2)$, $B \equiv (2, 4)$ and $C \equiv (-1, 3)$. Show that AB is perpendicular to AC. Let D be the mid-point of BC. Find the equation of the line AD.

8. It is given that circle $x^2 + y^2 = 8$ meets the line $x + y = k$, where $k \in \mathbb{R}$. Show that $-4 \leq k \leq 4$.

[see page six

9. A spherical balloon is being inflated such that its volume increases at a constant rate of $10\,\text{cm}^3\,\text{s}^{-1}$. Find the rate at which the radius is increasing when the radius is $10\,\text{cm}$.

10. Find the coordinates of the points on the curve $y = \frac{x^3}{3} - 2x^2$ at which the tangents are parallel to the line $y + 3x = 0$.

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[see page seven

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கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2017 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2017

ගණිතය I
கணிதம் I
Mathematics I

07 E I

PART B

* Answer **five** questions only.

11. (*a*) Let $0<m<9$. Show that the equation $mx^2+4(m+3)x+5m+19=0$ has two real distinct roots.

Let α and β be these roots. The roots of the equation $x^2+ax+b=0$ are $\alpha+\frac{1}{\alpha}$ and $\beta+\frac{1}{\beta}$, where $a, b\in\mathbb{R}$. Find a and b in terms of m.

(*b*) Let $f(x)=2x^3+3x^2+px-6$. It is given that $(x+3)$ is a factor of $f(x)$. Find the value of p. Also, find the remainder when $f(x)$ is divided by $(x+3)(x-1)$.

12. (*a*) Using the **Principle of Mathematical Induction**, prove that

$$\sum_{r=1}^{n} r(r+2)=\frac{n}{6}(n+1)(2n+7) \quad \text{for all } n\in\mathbb{Z}^+.$$

(*b*) Let $f(r)=\frac{1}{4r^2}$ and $U_r=\frac{Ar+B}{r^2(r+1)^2}$ for $r\in\mathbb{Z}^+$, where $A, B\in\mathbb{R}$. Find the values of A and B such that $U_r=f(r)-f(r+1)$ for $r\in\mathbb{Z}^+$.

Hence find $\sum_{r=1}^{n} U_r$ and show that $\sum_{r=1}^{\infty} U_r$ is convergent.

Let $V_r=U_r+r(r+2)$ for $r\in\mathbb{Z}^+$.

Using the result in (*a*), find $\sum_{r=1}^{n} V_r$ and show that $\sum_{r=1}^{\infty} V_r$ is divergent.

13. (*a*) How many different four-digit numbers can be formed by selecting digits from the seven digits 1, 2, 3, 4, 5, 6, 7

(i) with repetitions,

(ii) without repetitions?

In each of the cases (i) and (ii), how many of the numbers formed are even?

(*b*) It is required to select a team of four students from a group consisting of eight girls and two boys. Find the number of different teams that can be selected if

(i) both boys are in the team,

(ii) only one boy is in the team,

(iii) both boys are not in the team.

[see page eight

14. (*a*) Let $\mathbf{A}=\begin{pmatrix}1 & 2 & 0\\ 0 & 3 & -1\end{pmatrix}$, $\mathbf{B}=\begin{pmatrix}1 & 0 & 0\\ 1 & 1 & 2\end{pmatrix}$ and $\mathbf{C}=\begin{pmatrix}1 & 0\\ 0 & 2\\ -1 & 1\end{pmatrix}$.

Find $\mathbf{A}+2\mathbf{B}$, $\mathbf{AC}$ and $\mathbf{BC}$, and verify that

(i) $(\mathbf{A}+2\mathbf{B})\mathbf{C} = \mathbf{AC}+2\mathbf{BC}$, and

(ii) $(\mathbf{AC})\mathbf{B} = \mathbf{A}(\mathbf{CB})$.

(*b*) Let $\mathbf{P}=\begin{pmatrix}1 & 1\\ 1 & 2\end{pmatrix}$ and $\mathbf{D}=\begin{pmatrix}3 & 0\\ 0 & 2\end{pmatrix}$. Write down $\mathbf{P}^{-1}$.

Find the 2×2 matrix $\mathbf{A}$ such that $\mathbf{AP}=\mathbf{PD}$.

15. (*a*) Find the binomial expansion of $(2+3x)^5$, simplifying the coefficients.

Hence, show that $(2+3x)^5+(2-3x)^5=a+bx^2+cx^4$, where a, b and c are constants to be determined.

Deduce the value of $2.03^5+1.97^5$.

(*b*) A person opens a bank account which pays 5% annual interest compounded monthly, by depositing Rs. 20 000 on 01st January 2 000. He then deposited Rs. 20 000 into this account on the first of every month for the next five years. Assuming that no other transactions were made during this period, find the balance in the account after 5 years.

16. Find the centre and the radius of the circle C_1 with the equation $x^2+y^2-4x-2y=20$.

Show that $P\equiv(5,5)$ is a point on C_1 and find the equation of the tangent l to C_1 at the point P.

The line, l meets the x-axis at the point Q. Find the equation of the circle C_2 with P and Q as the ends of a diameter.

Also, find the length of the common chord of C_1 and C_2.

17. (*a*) Find the value of $\lim\limits_{x\to 3}\dfrac{x^2-9}{x^2-2x-3}$.

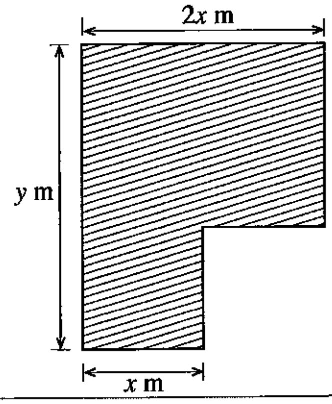
(*b*) Differentiate each of the following functions with respect to x:

(i) $\dfrac{x+1}{\sqrt{x^2+1}}$

(ii) $xe^{2x^2}+\dfrac{2}{e^{2x}}$

(iii) $\ln\left(x+\dfrac{1}{x}\right)$

(*c*) The shaded region of the figure shows a garden with the total perimeter 20 metres. It is constructed by removing a square of side x metres from a corner of a rectangular land of length $2x$ metres and width y metres. Find the value of x which makes the area of the garden a maximum.



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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2017 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2017 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2017

ගණිතය II
கணிதம் II
Mathematics II

07 E II

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

Index Number							

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- *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*
- *Statistical tables will be provided.*

For Examiners' Use only

(07) Mathematics II		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
	Percentage	

Paper **I**	
Paper **II**	
Total	
Final Marks	

Final Marks

In Numbers	
In Letters	

Code Numbers

Marking Examiner 1.	
Marking Examiner 2.	
Marks Checked by	
Supervised by	

[see page two

Part A

1. Find all real values of x satisfying the inequality $\frac{(x-1)(x-2)}{(x-3)} \geq 0$.

2. Shade the region in the xy-plane satisfying the inequalities $x^2+y^2 \leq 4$, $x+y \leq 2$, $y \geq -1$ and $-1 \leq x \leq 1$.

[see page three

3. Express $\cos x + \sqrt{3}\sin x$ in the form $R\cos(x-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Hence, solve the equation $\cos x + \sqrt{3}\sin x = 2\cos\left(x + \frac{\pi}{6}\right)$.

4. Using **integration by parts**, evaluate $\int_1^2 (2x+3)\ln x \, dx$.

[*see page four*

5. Express $\frac{1}{x(x-1)}$ in partial fractions. **Hence**, find $\int \frac{1}{x(x-1)} \mathrm{d}x$.

6. The probability distribution of a discrete random variable X is given below:

x	0	1	2
$P(X=x)$	k	k^2	k^2

Find the value of k and $E(X^2)$.

[see page five

7. A bookshop has two photocopy machines. In a certain week, the probability that both machines will break down is $\frac{1}{10}$ and the probability that only one machine will break down is $\frac{1}{3}$. Assume that the machines perform independently. Find the probability that both machines will work properly in that week.

8. Let A and B be two independent events such that $P(A)=0.1$ and $P(B)=0.6$. Find $P(A \cup B)$, $P(A' \cap B')$ and $P(A'|B')$.

[see page six

9. In a population, 10% is left-handed. If four persons are randomly selected, find the probability that at least one of them is left-handed.

Given that at least one of them is left-handed, find the probability that two of them are left-handed.

10. In a certain city, the probability that a sunny day is followed by another sunny day is 0.8 and the probability that a rainy day is followed by another rainy day is 0.5.

(i) Write down the one-step transition probability matrix.

(ii) If today is sunny, find the probability that day after tomorrow will also be sunny.

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[*see page seven*

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General Certificate of Education (Adv. Level) Examination, August 2017

ගණිතය II
கணிதம் II
Mathematics II

07 E II

PART B

* Answer **five** questions only.

11. A factory manufactures two products, 1 and 2, using three types of machines A, B and C. Both products must go to each machine and in the following order:

First to A, then to B and then to C.

The following table gives the hours needed at each machine per unit and the available hours for each machine per week:

Machine Type	Hours needed for		Available hours per week
	Product 1	Product 2	
A	2	2	16
B	1	2	12
C	4	2	28

The profit per unit of product 1 and product 2 are Rs. 10 000 and Rs. 15 000 respectively.

(i) Formulate this as a linear programming problem.

(ii) Sketch the feasible region.

(iii) Find the number of units of each product that would maximize the profit per week.

12. (*a*) Find the solutions of the equation $\dfrac{3+\sin^2\theta}{\cos\theta-2}=3\cos\theta$ in the range $0\le\theta<2\pi$.

(*b*) Show that $\tan^{-1}x+\tan^{-1}\dfrac{1}{x}=\dfrac{\pi}{2}$ for $x>0$.

(*c*) In a triangle ABC, $AB=6$ cm, $BC=7$ cm and $CA=5$ cm. Using the Cosine Rule for the triangle ABC, show that $\cos A=\dfrac{1}{5}$ and find its area.

13. (*a*) Using the substitution $u=(x-1)^2$, find $\int x(x-2)(x-1)^3\,dx$.

(*b*) The table below gives the values of the function $f(x)=\dfrac{1}{x^2-1}$ for x values between 2 and 3 with interval length 0.25, correct to three decimal places:

x	2	2.25	2.5	2.75	3
$f(x)$	0.333	0.246	0.190	0.152	0.125

Using Simpson's Rule, find an approximate value for $I=\displaystyle\int_2^3\frac{1}{x^2-1}\,dx$.

Hence, find an approximate value for $\ln\left(\dfrac{3}{2}\right)$.

[see page eight

14. In a set of n data values $x_i (i=1,2,\ldots,n)$ the mean and the standard deviation of the data are $\bar{x}$ and s respectively. Find the mean and the standard deviation of the new data values y obtained by the transformation $y=ax+b$.

The following frequency table summarises the number of milk packets sold by a certain cafeteria during 150 days:

Number of milk packets sold	Number of days
70 – 80	5
80 – 90	15
90 – 100	20
100 – 110	50
110 – 120	60

(i) Using the transformation $d_i = x_i - 95$ or otherwise, calculate the mean and the standard deviation of the frequency distribution summarised in the table.

(ii) Suppose on each of the above days, the cafeteria had received 120 packets to be sold. The profit from each packet sold is Rs. 15 and the loss from each unsold packet is Rs. 5. Find the mean of the net profit, of the milk packets received in the 150 days.

15. A survey carried out in a restaurant indicated that out of the customers who ordered juice, 50% had ordered orange juice, 20% had ordered apple juice and 30% had ordered lemon juice. The percentages of customers satisfied with the taste for each of orange juice, apple juice and lemon juice were 90%, 80% and 80% respectively.

(i) Find the probability that a randomly selected customer who ordered juice from this restaurant was satisfied with the taste.

(ii) Find the probability that a randomly selected customer had ordered orange juice and was satisfied with the taste.

(iii) If a customer is found to be **not** satisfied with the taste of the juice, find the probability that he had ordered lemon juice.

(iv) If a customer was randomly selected from those who ordered orange juice or lemon juice, find the probability that he was satisfied with the taste.

16. The length of A4 size sheets cut by a certain machine is normally distributed with a mean of 12 inches and a standard deviation of 1 inch. Sheets with length less than 11 inches or with lengths more than 13 inches are considered as unacceptable.

(i) Find the percentage of unacceptable sheets produced by the machine.

(ii) Given that a sheet produced by the machine is unacceptable, find the probability that the length of the sheet had exceeded 14 inches.

(iii) The company plans to reduce the standard deviation to maintain the unacceptable percentage of sheets within 1%. Find the largest possible standard deviation that meets this requirement.

17. A random variable X that assumes only non-negative values has a probability density function with an exponential distribution given by $f(x) = \lambda e^{-\lambda x}$, where $\lambda (>0)$ is a parameter. Show that the mean of the random variable is $\frac{1}{\lambda}$.

The life time of a certain electric equipment follows an exponential distribution with a mean life time of 2 years.

(i) Find the probability that an equipment fails before one year. (Take $e^{-0.5} \approx 0.6065$)

(ii) Determine the warranty period so that only 2% of the equipment fails within the warranty period.

(iii) A person bought two of the above electric equipment. Find the probability that at least one of the equipment fails before one year.

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