

**தமிழ்நாடு கல்வித் துறை - புதிய/பழைய பாடத்திட்டம் - New/Old Syllabus**

**NEW/OLD**

Department of Examinations, Sri Lanka

**අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු**  
**கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஆகஸ்ட்**  
**General Certificate of Education (Adv. Level) Examination, August 2019**

උසස් ගණිතය I  
 உயர் கணிதம் I  
**Higher Mathematics I**

**11 E I**

**28.08.2019 / 0830 - 1140**

භාග්‍ය භාග්‍ය  
 மூன்று மணித்தியாலம்  
**Three hours**

අමතර කියවීමේ කාලය - මිනිත්තු 10 යි  
 மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்  
**Additional Reading Time - 10 minutes**

Use **additional reading time** to go through the question paper, select the questions and decide on the questions that you give priority in answering.

**Instructions:**

Index Number

- \* This question paper consists of two parts;  
**Part A** (Questions 1 - 10) and **Part B** (Questions 11 - 17).
- \* **Part A:**  
 Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
- \* **Part B:**  
 Answer **five** questions only. Write your answers on the sheets provided.
- \* At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.
- \* You are permitted to remove **only Part B** of the question paper from the Examination Hall.

**For Examiners' Use only**

(11) Higher Mathematics I		
Part	Question No.	Marks
A	1	
	2	
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B	11	
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	17	
	<b>Total</b>	

**Total**

In Numbers

In Words

**Code Numbers**

Marking Examiner

Checked by:

Supervised by:

1. Factorize:  $a^2(b-c)^3 + b^2(c-a)^3 + c^2(a-b)^3$ .

- [see page three

- Write down the range of  $f$  and find  $f^{-1}(x)$ . Also, find  $f(2f^{-1}(0))$ .

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- $$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

More Past Papers at  
**tamilguru.lk**

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- Show also that the area of the triangle  $OAP$  is  $3a^2$ , where  $O$  is the origin.

[illegible]

- $$f(x) = \begin{cases} a\left(1 + e^{-\frac{1}{x}}\right) & \text{if } x > 0, \\ 2 & \text{if } x = 0, \\ \frac{\sqrt{1+bx}-1}{x} & \text{if } x < 0. \end{cases}$$

It is given that  $f(x)$  is continuous at  $x = 0$ . Find the values of  $a$  and  $b$ .

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Is  $f'(x)$  differentiable at  $x = 1$ ?

[illegible]This image shows a full page of white paper with horizontal dotted lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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- This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- [illegible]

## உல/புரலி கிரேடிய - புதிய/பழைய பாடத்திட்டம் - New/Old Syllabus

**NEW/OLD**இலங்கை தேசிய தேர்வுகள்  
Department of Examinations, Sri Lankaඅධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු  
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஓகஸ்ட்  
General Certificate of Education (Adv. Level) Examination, August 2019උසස් ගණිතය I  
உயர் கணிதம் I  
Higher Mathematics I

11 E I

## Part B

\* Answer five questions only.

11.(a) Let  $A$ ,  $B$  and  $C$  be subsets of a universal set  $S$ . Stating clearly any result in Algebra of Sets that you use, show that

(i)  $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$ ,

(ii)  $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

Using an example, show that  $A \setminus (B \cup C) \neq (A \setminus B) \cup (A \setminus C)$ .

(b) In a competition, a school awarded medals for three different categories as follows:

45 medals were awarded for dancing,

21 medals were awarded for singing, and

27 medals were awarded for sports.

If these medals were awarded to a total of 54 recipients and only 13 persons received medals in all three categories, how many persons received medals, exactly in two of these categories?

12.(a) Let  $a, b, c \in \mathbb{R}^+$ .

Using Arithmetic Mean – Geometric Mean inequality, show that

$$\frac{a}{b} + \frac{b}{a} \geq 2.$$

Hence, show that

(i)  $\frac{b+c}{a} + \frac{c+a}{b} + \frac{a+b}{c} \geq 6$ ,

(ii)  $x^2(1+y^2) + y^2(1+z^2) + z^2(1+x^2) \geq 6xyz$ .

More Past Papers at  
**tamilguru.lk**(b) The transformation  $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ , maps points in the  $xy$ -plane into points in the  $x'y'$ -plane. Find the value of  $m$  in order that the line  $y=mx+c$ , ( $m \neq \frac{2}{3}$  and  $c \neq 0$ ), is invariant under the above transformation.Let  $A \equiv (c, 0)$  and  $B \equiv (0, c)$  be two points in the  $xy$ -plane. Find the coordinates of their images  $A'$  and  $B'$  under this transformation and verify that the points  $A'$  and  $B'$  lie on the line  $x' + y' = c$ .

13. State and prove **De Moivre's Theorem** for a positive integral index.

Using **De Moivre's Theorem**, show that

(i)  $\cos 5\theta = \cos^5 \theta - 10 \cos^3 \theta \sin^2 \theta + 5 \cos \theta \sin^4 \theta$ , and

(ii)  $\sin 5\theta = \sin^5 \theta - 10 \cos^2 \theta \sin^3 \theta + 5 \cos^4 \theta \sin \theta$ .

**Deduce** that  $\tan 5\theta = \frac{\tan \theta (\tan^4 \theta - 10 \tan^2 \theta + 5)}{(1 - 10 \tan^2 \theta + 5 \tan^4 \theta)}$ .

Solve the equation  $\tan 5\theta = 0$  for  $0 < \theta < \frac{\pi}{2}$ , and show that  $\tan^2\left(\frac{\pi}{5}\right)$  and  $\tan^2\left(\frac{2\pi}{5}\right)$  are the roots of the equation  $x^2 - 10x + 5 = 0$ .

**Hence**, show that  $\sec^2\left(\frac{\pi}{5}\right) + \sec^2\left(\frac{2\pi}{5}\right) = 12$ .

14.(a) Let  $C_1$  and  $C_2$  be the curves given by  $y = \frac{4x}{1+x}$  and  $y = \frac{2}{3}x^2$  for  $x \in \mathbb{R}$ , respectively. Find the coordinates of the points of intersection of  $C_1$  and  $C_2$ .

Sketch the graphs of  $C_1$  and  $C_2$  in the same figure showing clearly the asymptotes and turning points (if any). Find the area enclosed by  $C_1$  and  $C_2$ .

Find also the volume of the solid generated by revolving the area enclosed by the curves  $C_1$  and  $C_2$  through 4-right angles about the  $x$ -axis.

(b) Solve the differential equation:

$$2x^2 \frac{dy}{dx} - 2xy + y^2 = 0.$$

15.(a) Let  $I_n = \int_0^{2\pi} \sin^n(x+\alpha) dx$ , where  $\alpha$  is a real constant and  $n$  is an integer such that  $n \geq 2$ .

Show that,  $nI_n = (n-1)I_{n-2}$  for  $n \geq 2$ .

**Hence**, find the value of  $\int_0^{2\pi} (\sqrt{3} \sin x + \cos x)^6 dx$ .

(b) Let  $y = \tan(e^{2x} - 1)$ .

Show that  $\frac{d^2y}{dx^2} = 2 \frac{dy}{dx} (1 + ye^{2x})$ .

**Hence**, find the Maclaurin series expansion of  $y$  up to and including the term involving  $x^4$ .

16. Show that the equation of the tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  at the point  $(x_1, y_1)$  is

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

**Deduce** the equation of the tangent to the ellipse at the point  $P$  with eccentric angle  $\theta$ , and show that the normal to the ellipse at  $P$  is given by  $(a \sec \theta)x - (b \operatorname{cosec} \theta)y = a^2 - b^2$ .

Let  $T$  and  $T'$  be the points where the tangent meets the  $OX$  and  $OY$  axes, respectively, and let  $N$  and  $N'$  be the points where the normal meets the  $OX$  and  $OY$  axes, respectively.

(i) Show that the equation of the locus of the mid-point of  $NN'$  as  $\theta$  varies is  $4(a^2x^2 + b^2y^2) = (a^2 - b^2)^2$ .

(ii) Find the value of the eccentric angle  $\theta$  ( $0 < \theta < \frac{\pi}{2}$ ) at which the lines  $TT'$  and  $NN'$  are equally inclined to the two axes of coordinates. In this case, find  $(TT')(NN')$  in terms of  $a$  and  $b$ .



17.(a) Let  $f(x) = \frac{\sin 2x}{2 + \cos 2x}$  for  $x \in \mathbb{R}$ .

(i) Show that  $-\frac{1}{\sqrt{3}} \leq f(x) \leq \frac{1}{\sqrt{3}}$  for  $x \in \mathbb{R}$ .

(ii) Sketch the graph of  $y = f(x)$  for  $0 \leq x \leq \pi$ .

(b) Using **Simpson's Rule** with the values of  $e^{-x^2}$  given in the following table, find an approximate value for  $\int_0^1 e^{-x^2} dx$ :

$x$	0	0.25	0.50	0.75	1.0
$e^{-x^2}$	1	0.9394	0.7788	0.5698	0.3679

**Deduce** an approximate value for  $\int_0^1 e^{(\ln 2 - 9x^2)} dx$ .

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## නව/පැරණි තීර්ථය - ප්‍රதிய/පழைய පාලනීය - New/Old Syllabus

NEW/OLD

Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු  
 கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஆகஸ்ட்  
 General Certificate of Education (Adv. Level) Examination, August 2019

උසස් ගණිතය II  
 உயர் கணிதம் II  
 Higher Mathematics II

11 E II

31.08.2019 / 1300 - 1610

දැය නූතනී  
 மூன்று மணித்தியாலம்  
 Three hours

අමතර කියවීමේ කාලය - මිනිත්තු 10 පි  
 மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்  
 Additional Reading Time - 10 minutes

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- \* You are permitted to remove **only Part B** of the question paper from the Examination Hall.
- \* Statistical Tables will be provided.
- \*  $g$  denotes the acceleration due to gravity.

## For Examiners' Use only

(11) Higher Mathematics II		
Part	Question No.	Marks
A	1	
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B	11	
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	Total	

Total

In Numbers

In Words

Code Numbers

Marking Examiner

Checked by:

1

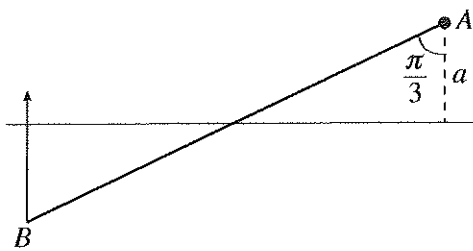
2

Supervised by:

1. The position vectors of three points  $A, B$  and  $C$  with respect to a fixed origin  $O$  are  $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ ,  $2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$  and  $7\mathbf{i} + \alpha\mathbf{j} + \beta\mathbf{k}$ , respectively. Find the values of the constants  $\alpha$  and  $\beta$  such that the vector  $\vec{OC}$  is perpendicular to the plane  $OAB$ .

2. A force  $\mathbf{F}$  of magnitude 6 N acts along the line whose vector equation is  $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 4\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ , where  $\lambda$  is a scalar parameter. If the distances are measured in metres, show that the vector moment  $\mathbf{M}$  of  $\mathbf{F}$  about the origin is of magnitude  $4\sqrt{5}$  N m and that  $\mathbf{M} \cdot \mathbf{k} = 0$ .

3. A uniform rod  $AB$  of length  $4a$  and density  $\rho$  is smoothly hinged at the end  $A$  to a fixed point at a height  $a$  above the free surface of a homogeneous liquid of density  $\sigma \left( < \frac{4\rho}{3} \right)$ . The rod is kept in equilibrium, making an angle  $\frac{\pi}{3}$  with the downward vertical as shown in the figure, by a vertical light inextensible string attached to the end  $B$ . Find the tension in the string.



4. With respect to a fixed origin, the position vector  $\mathbf{r}$  of a particle  $P$  at time  $t$  is given by  $\mathbf{r} = a(\omega t - \sin \omega t) \mathbf{i} + a(\omega t - \cos \omega t) \mathbf{j}$ , where  $a$  and  $\omega$  are positive constants and  $0 \leq \omega t \leq \pi$ . Find the velocity vector  $\mathbf{v}$  and the acceleration vector  $\mathbf{f}$  of  $P$ , at time  $t$ . Find the time at which  $\mathbf{v} \cdot \mathbf{f} = 0$ , and show that the speed of  $P$  at this instant is  $a\omega(\sqrt{2}-1)$ .

- [illegible]

- 

[illegible]

- [illegible]

- [illegible]

Find the values of constants  $k$  and  $a$  in order that  $f(x)$  is the probability density function of a continuous random variable  $X$  whose mean is  $\frac{8}{15}$ . Show that the standard deviation of  $X$  is  $\frac{\sqrt{11}}{15}$ .

10. The cumulative distribution function  $F(x)$  of a discrete random variable  $X$  is given by  $F(x) = \frac{1}{16}(8x - x^2)$  for  $x = 1, 2, 3, 4$ . Obtain the probability mass function of  $X$  and find  $E(X)$ .

## නව/පැරණි නිර්දේශය - புதிய/பழைய பாடத்திட்டம் - New/Old Syllabus

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව  
 இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம்  
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 ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව  
 இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரīட்சைத் திணைக்களம் இலங்கைப் பரīட்சைத் திணைக்களம் இலங்கைப் பரīட்சைத் திணைக்களம்  
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**NEW/OLD**

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු  
 கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஆகஸ்ட்  
 General Certificate of Education (Adv. Level) Examination, August 2019

උසස් ගණිතය II  
 உயர் கணிதம் II  
**Higher Mathematics II**

**11 E II**

**Part B**\* Answer **five** questions only.

11. A system consists of three forces acting at points having position vectors, with respect to an origin  $O$ , as given in the following table.

Point	Position vector	Force
$A_1$	$\mathbf{r}_1 = 2\mathbf{i} - 4\mathbf{j}$	$\mathbf{F}_1 = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$
$A_2$	$\mathbf{r}_2 = \mathbf{j} - 3\mathbf{k}$	$\mathbf{F}_2 = -3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$
$A_3$	$\mathbf{r}_3 = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$	$\mathbf{F}_3 = -\mathbf{i} - \mathbf{j} + 2\mathbf{k}$

By introducing forces  $\pm \mathbf{F}_s$ ,  $s = 1, 2, 3$ , at the origin  $O$ , show that the given system can be reduced to a single force  $\mathbf{R} = \sum_{s=1}^3 \mathbf{F}_s$  acting at  $O$  together with a couple of vector moment  $\mathbf{G} = \sum_{s=1}^3 \mathbf{r}_s \times \mathbf{F}_s$ . Find vectors  $\mathbf{R}$  and  $\mathbf{G}$  in terms of  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$ .

**Deduce** that the system is equivalent to a single resultant force  $\mathbf{R}$  of magnitude  $\sqrt{26}$ .

Show that the lines of action of  $\mathbf{F}_1$  and  $\mathbf{F}_2$  meet at a certain point  $A_0$  with position vector  $\mathbf{r}_0$ , where  $\mathbf{r}_0$  is to be determined. Verify that the line of action of  $\mathbf{F}_3$  also passes through the point  $A_0$ .

Write down the equation of the line of action of the single resultant force  $\mathbf{R}$  in the form  $\mathbf{r} = \mathbf{r}_0 + \gamma \mathbf{R}$ .

Find the position vector of the point where this line meets the  $xy$ -plane.

**Hence**, show that the Cartesian equations of the line of action of the resultant force  $\mathbf{R}$  may be obtained as  $\frac{x-6}{3} = \frac{y+4}{-4} = \frac{z}{1}$ .

Show further that the Cartesian equation of the plane on which the given system of forces lies is obtained as  $x + 3z = 0$ .

12. A lamina in the shape of a trapezium  $ABCD$  with  $AB$  parallel to  $DC$ ,  $AB = 3a$ ,  $DC = a$  and  $\hat{BAD} = \hat{BC} = \frac{\pi}{4}$  is immersed vertically in a homogeneous liquid such that  $AB$  is on the free surface of the liquid.

Show that the centre of pressure of the lamina  $ABCD$  is at a distance  $\frac{3a}{5}$  vertically below the mid-point  $E$  of  $AB$ .

A door in the shape of the above lamina  $ABCD$  is made on a vertical side of a tank with  $AB$  horizontal and  $CD$  below  $AB$ . The door is smoothly hinged along  $CD$ . The tank is filled to the level of  $AB$  with a homogeneous liquid of density  $\rho$ .

Find the least force that should be applied at  $E$  in order to keep the door closed with liquid inside the tank.



13. An engine pulls a train along a straight horizontal track against a resistance which at any time is  $k$  times the momentum of the train, where  $k$  is a constant. The engine works at constant power  $9Mkv_0^2$ , where  $M$  is the total mass of the engine and the train. Show that

(i) the maximum speed that the train can attain is  $3v_0$ .

(ii) the time taken for the train to increase the speed from  $v_0$  to  $2v_0$  is  $\frac{1}{2k} \ln\left(\frac{8}{5}\right)$ .

When the train is moving with speed  $U$  its power is cut off and a constant braking force of magnitude  $F$  is applied, in addition to the above resistance. Show that the train will stop at a time  $\frac{1}{k} \ln\left(\frac{F + MkU}{F}\right)$  after the power is cut off.

14. A particle  $P$  of mass  $m$  resting on a smooth horizontal table is connected to a fixed point  $O$  on the table by a light elastic string of natural length  $a$  and modulus of elasticity  $mg$ . When time  $t = 0$ , the particle  $P$  is at a distance  $a$  from  $O$ , with the string just taut, and  $P$  is projected along the table in a direction perpendicular to the initial line of the string, with velocity of magnitude  $U = 2\sqrt{\frac{ga}{3}}$ .

Using the principle of conservation of energy and the principle of conservation of angular momentum about  $O$ , show that

$$\left(\frac{dr}{dt}\right)^2 = U^2 \left(1 - \frac{a^2}{r^2}\right) - \frac{g}{a}(r-a)^2.$$

**Deduce** that

(i) the maximum length of the string is  $2a$ , and the tension in the string at this instant is  $mg$ ,

(ii) the speed of the particle at this instant is  $\frac{U}{2}$ .

Find  $\frac{d^2r}{dt^2}$ , in terms of  $r$  and  $a$  when  $\frac{dr}{dt} \neq 0$ .

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15. Show that

(i) the moment of inertia of a uniform hollow circular cylinder, of mass  $M$  and radius  $a$  about its axis is  $Ma^2$ , and

(ii) the moment of inertia of a uniform circular disc of mass  $m$  and radius  $a$  about the axis through its centre perpendicular to its plane is  $\frac{1}{2}ma^2$ .

A closed container  $C$  made from a thin uniform metal sheet, consists of a right hollow circular cylinder of radius  $a$  and length  $3a$  to which two uniform circular discs, each of radius  $a$ , are attached at the two ends. Show that the radius of gyration,  $k$  of the container  $C$  about its axis is given by  $k^2 = \frac{7}{8}a^2$ .

The container rolls, without slipping, down a rough plane of inclination  $\alpha$  to the horizontal, with its axis horizontal and perpendicular to the lines of greatest slope of the plane.

Show that the acceleration  $f$  of the container  $C$ , in this motion, is given by  $f = \frac{8}{15}g \sin \alpha$ , and that the coefficient of friction  $\mu$  between the container and the plane is such that  $\mu > \frac{8}{15} \tan \alpha$ .

- 16.(a) Let  $X$  be the number of vehicles leaving a certain car park during a five minutes interval. Suppose  $X$  has the following probability distribution:

$x$	1	2	3	4	5	6
$P(X=x)$	$p$	$2p$	$3p$	$3p$	$2p$	$p$

Find the value of  $p$  and the expected value  $E(X)$  of  $X$ .

Show that the standard deviation of  $X$  is  $\frac{\sqrt{7}}{2}$ .

The random variable  $Y$  is defined by  $Y = 2X + 3$ . Find the expected value  $E(Y)$  of  $Y$  and the standard deviation of  $Y$ .

Also, find the value of  $P(Y \geq E(Y))$ .

- (b) The probability that a patient recovers from a delicate surgery is  $\frac{2}{5}$ . Five patients were monitored at random, after they underwent this surgery.

Find the probability that

- (i) at least 3 recover,
- (ii) exactly 2 recover,
- (iii) none recovers.

- 17.(a) The lifetime,  $T$  hours, of a certain kind of electric lamp can be modelled by the probability density function

$$f(t) = \begin{cases} \frac{1}{a} e^{-\left(\frac{1}{b}\right)t}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

where  $a$  and  $b$  are positive constants.

Show that  $a = b$ .

It is given that 40% of lamps have a lifetime longer than 2000 hours. Find the common value of  $a$  and  $b$ .

Find the distribution function of  $T$ , and **hence**, show that  $P(T > t + c | T > c) = P(T > t)$ , where  $t \geq 0$  and  $c$  is a positive constant.

- (b) The speeds of vehicles passing a certain point A on a motorway can be taken to be normally distributed. Observations show that of vehicles passing the point A, 95% are travelling at speed less than  $85 \text{ km h}^{-1}$  and 10% are travelling at less than  $55 \text{ km h}^{-1}$ .

- (i) Find the average speed of the vehicles passing the point A.
- (ii) Find the percentage of vehicles that travel at a speed more than  $70 \text{ km h}^{-1}$ .

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