

நல/உருளி திரட்டை - புதிய/பழைய பாடத்திட்டம் - *New/Old Syllabus*

# NEW/OLD

Department of Examinations, Sri Lanka

අධ්‍යාපන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු කළුවිප් පොතුත් තරාතරුප පත්තිර (ශ්‍යර් තු)ප පරිශ්‍යා සහ, 2019 ඉකස්ස් අධ්‍යාපන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු කළුවිප් පොතුත් තරාතරුප පත්තිර (ශ්‍යර් තු)ප පරිශ්‍යා සහ, 2019 ඉකස්ස් General Certificate of Education (Adv. Level) Examination, August 2019

# உயர் கணிதம்

## Higher Mathematics

11 E I

28.08.2019 / 0830 – 1140

ஏட ஏந்தி  
மூன்று மணித்தியாலம்  
*Three hours*

அமுலர் தியலீலி காலை	- மீனித்து 10 டி
மேலதிக வாசிப்பு நேரம்	- 10 நிமிடங்கள்
<b>Additional Reading Time</b>	<b>10 minutes</b>

**Use additional reading time** to go through the question paper, select the questions and decide on the questions that you give priority in answering.

### Instructions:

Index Number						
--------------	--	--	--	--	--	--

- \* *This question paper consists of two parts; Part A (Questions 1 - 10) and Part B (Questions 11 - 17).*
- \* **Part A:**  
*Answer all questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
- \* **Part B:**  
*Answer five questions only. Write your answers on the sheets provided.*
- \* *At the end of the time allotted, tie the answer scripts of the two parts together so that Part A is on top of Part B and hand them over to the supervisor.*
- \* *You are permitted to remove only Part B of the question paper from the Examination Hall.*

**For Examiners' Use only**

(11) Higher Mathematics I		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

<b>Total</b>	
In Numbers	
In Words	

Code Numbers	
Marking Examiner	
Checked by:	1
	2
Supervised by:	

## Part A

1. Factorize:  $a^2(b-c)^3 + b^2(c-a)^3 + c^2(a-b)^3$ .

2. Let  $R$  be the relation defined on the set of integers  $\mathbb{Z}$  by  $xRy$  if 7 divides  $(x^3 - y^3)$ . Show that  $R$  is an equivalence relation on  $\mathbb{Z}$  and write down the equivalence class of 0.

**Index Number**

3. Let  $f(x) = \frac{x+2}{x-3}$  for  $x \neq 3$ .

Write down the range of  $f$  and find  $f^{-1}(x)$ . Also, find  $f(2f^{-1}(0))$ .

4. Show that

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

More Past Papers at  
**tamilguru.lk**

5. Show that the line  $y = \frac{1}{3}x + 3a$  touches the parabola  $y^2 = 4ax$ , at a point  $P$  such that  $AP = 10a$ , where  $A$  is the focus of the parabola.

Show also that the area of the triangle  $OAP$  is  $3a^2$ , where  $O$  is the origin.

6. Let  $a, b \in \mathbb{R}$  and let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be the function given by

$$f(x) = \begin{cases} a \left( 1 + e^{-\frac{1}{x}} \right) & \text{if } x > 0, \\ 2 & \text{if } x = 0, \\ \frac{\sqrt{1+bx}-1}{x} & \text{if } x < 0. \end{cases}$$

It is given that  $f(x)$  is continuous at  $x = 0$ . Find the values of  $a$  and  $b$ .

$$7. \text{ Let } f(x) = \begin{cases} x^2 + 3x + 3 & \text{if } x \leq 1, \\ 5x + 2 & \text{if } x > 1. \end{cases}$$

Show that  $f(x)$  is differentiable at  $x = 1$  and write down  $f'(x)$  for  $x \in \mathbb{R}$ .

Is  $f'(x)$  differentiable at  $x = 1$ ?

8. Solve the differential equation  $\frac{dy}{dx} - y \cot x + 3\sin^2 2x = 0$ , subject to the condition that  $y=1$  when  $x = \frac{\pi}{4}$ .

9. Let  $f$  and  $g$  be real valued functions defined on the interval  $[0, 2]$  such that  $f'$  and  $g$  are both continuous on  $[0, 2]$  and  $xf'(x) = g(2 - x)$  for all  $x \in [0, 2]$ . If  $f(2) = 1$  and  $\int_0^2 f(x)dx = 3$ , find  $\int_0^2 g(x)dx$ .

10. Sketch the curves given in polar coordinates by  $r = 2 \cos \theta$  and  $r(\cos \theta + \sin \theta) = 1$  in the same diagram. Find the polar coordinates of their points of intersection.



13. State and prove **De Moivre's Theorem** for a positive integral index.

Using **De Moivre's Theorem**, show that

- (i)  $\cos 5\theta = \cos^5 \theta - 10 \cos^3 \theta \sin^2 \theta + 5 \cos \theta \sin^4 \theta$ , and
- (ii)  $\sin 5\theta = \sin^5 \theta - 10 \cos^2 \theta \sin^3 \theta + 5 \cos^4 \theta \sin \theta$ .

Deduce that  $\tan 5\theta = \frac{\tan \theta (\tan^4 \theta - 10 \tan^2 \theta + 5)}{(1 - 10 \tan^2 \theta + 5 \tan^4 \theta)}$ .

Solve the equation  $\tan 5\theta = 0$  for  $0 < \theta < \frac{\pi}{2}$ , and show that  $\tan^2\left(\frac{\pi}{5}\right)$  and  $\tan^2\left(\frac{2\pi}{5}\right)$  are the roots of the equation  $x^2 - 10x + 5 = 0$ .

Hence, show that  $\sec^2\left(\frac{\pi}{5}\right) + \sec^2\left(\frac{2\pi}{5}\right) = 12$ .

14.(a) Let  $C_1$  and  $C_2$  be the curves given by  $y = \frac{4x}{1+x}$  and  $y = \frac{2}{3}x^2$  for  $x \in \mathbb{R}$ , respectively. Find the coordinates of the points of intersection of  $C_1$  and  $C_2$ .

Sketch the graphs of  $C_1$  and  $C_2$  in the same figure showing clearly the asymptotes and turning points (if any). Find the area enclosed by  $C_1$  and  $C_2$ .

Find also the volume of the solid generated by revolving the area enclosed by the curves  $C_1$  and  $C_2$  through 4-right angles about the  $x$ -axis.

(b) Solve the differential equation:

$$2x^2 \frac{dy}{dx} - 2xy + y^2 = 0.$$

15.(a) Let  $I_n = \int_0^{2\pi} \sin^n(x+\alpha) dx$ , where  $\alpha$  is a real constant and  $n$  is an integer such that  $n \geq 2$ .

Show that,  $nI_n = (n-1)I_{n-2}$  for  $n \geq 2$ .

Hence, find the value of  $\int_0^{2\pi} (\sqrt{3} \sin x + \cos x)^6 dx$ .

(b) Let  $y = \tan(e^{2x} - 1)$ .

Show that  $\frac{d^2y}{dx^2} = 2 \frac{dy}{dx} (1 + ye^{2x})$ .

Hence, find the Maclaurin series expansion of  $y$  up to and including the term involving  $x^4$ .

16. Show that the equation of the tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  at the point  $(x_1, y_1)$  is

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1.$$

Deduce the equation of the tangent to the ellipse at the point  $P$  with eccentric angle  $\theta$ , and show that the normal to the ellipse at  $P$  is given by  $(a \sec \theta)x - (b \cosec \theta)y = a^2 - b^2$ .

Let  $T$  and  $T'$  be the points where the tangent meets the  $OX$  and  $OY$  axes, respectively, and let  $N$  and  $N'$  be the points where the normal meets the  $OX$  and  $OY$  axes, respectively.

(i) Show that the equation of the locus of the mid-point of  $NN'$  as  $\theta$  varies is  $4(a^2x^2 + b^2y^2) = (a^2 - b^2)^2$ .

(ii) Find the value of the eccentric angle  $\theta$  ( $0 < \theta < \frac{\pi}{2}$ ) at which the lines  $TT'$  and  $NN'$  are equally inclined to the two axes of coordinates. In this case, find  $(TT')(NN')$  in terms of  $a$  and  $b$ .

17.(a) Let  $f(x) = \frac{\sin 2x}{2 + \cos 2x}$  for  $x \in \mathbb{R}$ .

(i) Show that  $-\frac{1}{\sqrt{3}} \leq f(x) \leq \frac{1}{\sqrt{3}}$  for  $x \in \mathbb{R}$ .

(ii) Sketch the graph of  $y = f(x)$  for  $0 \leq x \leq \pi$ .

(b) Using Simpson's Rule with the values of  $e^{-x^2}$  given in the following table, find an approximate value for  $\int_0^1 e^{-x^2} dx$ :

$x$	0	0.25	0.50	0.75	1.0
$e^{-x^2}$	1	0.9394	0.7788	0.5698	0.3679

Deduce an approximate value for  $\int_0^1 e^{(\ln 2-9x^2)} dx$ .

\*\*\*

நில/பரவி திருட்டை - புதிய/பழைய பாடத்திட்டம் - New/Old Syllabus

# NEW/OLD ඩොෂ්‍යා දෙපාර්තමේන්තුව

අධ්‍යාපන පොදු සහකික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු කළුවිප් පොතුත් තරාතුරුප පත්තිර (ශ්‍යාරු තරු)ප රෝට්ස්, 2019 ඉකළුව් අධ්‍යාපන පොදු සහකික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු කළුවිප් පොතුත් තරාතුරුප පත්තිර (ශ්‍යාරු තරු)ප රෝට්ස්, 2019 ඉකළුව් General Certificate of Education (Adv. Level) Examination, August 2019

## உயர் கணிதம்

11 E II

31.08.2019 / 1300 - 1610

ஏடு இங்கி  
மூன்று மணித்தியாலம்  
**Three hours**

අමතර තියවේම කාලය	- මිනිත්තු 10 දි
මෙළතික බාසිපු තුරුම	- 10 නිමිටඛක්
<b>Additional Reading Time</b>	<b>10 minutes</b>

Use **additional reading time** to go through the question paper, select the questions and decide on the questions that you give priority in answering.

### Instructions:

**Index Number** \_\_\_\_\_

- \* This question paper consists of two parts; **Part A** (Questions 1 - 10) and **Part B** (Questions 11 - 17).
- \* **Part A:**  
Answer *all* questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
- \* **Part B:**  
Answer *five* questions only. Write your answers on the sheets provided.
- \* At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.
- \* You are permitted to remove **only Part B** of the question paper from the Examination Hall.
- \* Statistical Tables will be provided.
- \*  $g$  denotes the acceleration due to gravity.

**For Examiners' Use only**

(11) Higher Mathematics II		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

	<b>Total</b>
In Numbers	
In Words	

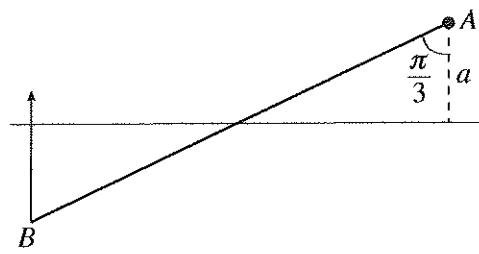
Code Numbers	
Marking Examiner	
Checked by:	1
	2
Supervised by:	

## Part A

1. The position vectors of three points  $A$ ,  $B$  and  $C$  with respect to a fixed origin  $O$  are  $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ ,  $2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$  and  $7\mathbf{i} + \alpha\mathbf{j} + \beta\mathbf{k}$ , respectively. Find the values of the constants  $\alpha$  and  $\beta$  such that the vector  $\overrightarrow{OC}$  is perpendicular to the plane  $OAB$ .

2. A force  $\mathbf{F}$  of magnitude 6 N acts along the line whose vector equation is  $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 4\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ , where  $\lambda$  is a scalar parameter. If the distances are measured in metres, show that the vector moment  $\mathbf{M}$  of  $\mathbf{F}$  about the origin is of magnitude  $4\sqrt{5}$  N m and that  $\mathbf{M} \cdot \mathbf{k} = 0$ .

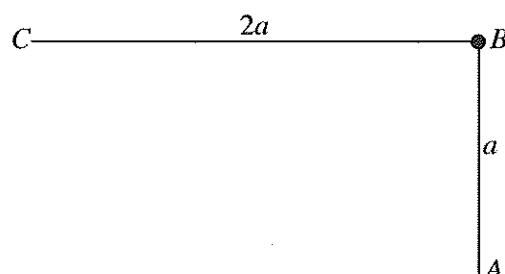
3. A uniform rod  $AB$  of length  $4a$  and density  $\rho$  is smoothly hinged at the end  $A$  to a fixed point at a height  $a$  above the free surface of a homogeneous liquid of density  $\sigma \left( < \frac{4\rho}{3} \right)$ . The rod is kept in equilibrium, making an angle  $\frac{\pi}{3}$  with the downward vertical as shown in the figure, by a vertical light inextensible string attached to the end  $B$ . Find the tension in the string.



4. With respect to a fixed origin, the position vector  $\mathbf{r}$  of a particle  $P$  at time  $t$  is given by  $\mathbf{r} = a(\omega t - \sin \omega t) \mathbf{i} + a(\omega t - \cos \omega t) \mathbf{j}$ , where  $a$  and  $\omega$  are positive constants and  $0 \leq \omega t \leq \pi$ . Find the velocity vector  $\mathbf{v}$  and the acceleration vector  $\mathbf{f}$  of  $P$ , at time  $t$ . Find the time at which  $\mathbf{v} \cdot \mathbf{f} = 0$ , and show that the speed of  $P$  at this instant is  $a\omega(\sqrt{2}-1)$ .

5. Two small smooth spheres  $P$  and  $Q$  of masses  $m$  and  $2m$  respectively, are moving on a smooth horizontal table and collide with each other. The velocities of  $P$  and  $Q$ , just before impact are  $2\mathbf{i} + \mathbf{j}$  and  $-2\mathbf{i} + 3\mathbf{j}$  respectively. The coefficient of restitution between  $P$  and  $Q$  is  $\frac{1}{3}$ . Write down equations sufficient to determine the velocities of  $P$  and  $Q$  just after impact.

6. A frame is formed by rigidly attaching a uniform rod  $AB$  of mass  $m$  and length  $a$  to a uniform rod  $BC$  of mass  $2m$  and length  $2a$ , in such a way that angle  $A\hat{B}C$  is a right angle. The frame can rotate freely about a fixed smooth horizontal axis through  $B$  and perpendicular to the plane of the frame. The frame is released from rest in the position in which  $BC$  is horizontal and  $A$  is below  $B$ . Assuming that the moment of inertia of the frame  $ABC$  about the axis of rotation is  $3ma^2$ , find the angular velocity of the frame when the rod  $BC$  is vertical with  $C$  below  $B$ .



7. A box contains 10 red and 15 green balls which are identical in all aspects other than colour. Balls are drawn from this box at random, one by one, with replacement.

(i) Calculate the probability that the first red ball is drawn on or before the 3<sup>rd</sup> draw.

(ii) Calculate the conditional probability of drawing first green ball on the 8<sup>th</sup> draw, given that first 5 balls drawn are red.

8. The number of misprints per page of a certain document follows a Poisson distribution with mean 2.1. Find the probability that a randomly selected page has

- (i) exactly 1 misprint,
- (ii) at least 3 misprints.

$$9. \text{ Let } f(x) = \begin{cases} kx(a-x^2), & 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the values of constants  $k$  and  $a$  in order that  $f(x)$  is the probability density function of a continuous random variable  $X$  whose mean is  $\frac{8}{15}$ . Show that the standard deviation of  $X$  is  $\frac{\sqrt{11}}{15}$ .

10. The cumulative distribution function  $F(x)$  of a discrete random variable  $X$  is given by  $F(x) = \frac{1}{16}(8x - x^2)$  for  $x = 1, 2, 3, 4$ . Obtain the probability mass function of  $X$  and find  $E(X)$ .

நல/பரவி திரட்டையை - புதிய/பழைய பாடக்குட்டம் - New/Old Syllabus

අධ්‍යාපන පොදු සහතික පත්‍ර (ලසක් පෙළ) විභාගය, 2019 අගෝස්තු කළමනිප් පොතුත් තරාතරුප පත්තිර (ශ්‍යාරු තරු)ප පරිශ්‍යෙ, 2019 ගුකළු අධ්‍යාපන පොදු සහතික පත්‍ර (Adv. Level) Examination, August 2019

**உயர் கணிதம்** **II**  
**Higher Mathematics** **II**

11 E II

## Part B

\* Answer **five** questions only.

11. A system consists of three forces acting at points having position vectors, with respect to an origin  $O$ , as given in the following table.

Point	Position vector	Force
$A_1$	$\mathbf{r}_1 = 2\mathbf{i} - 4\mathbf{j}$	$\mathbf{F}_1 = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$
$A_2$	$\mathbf{r}_2 = \mathbf{j} - 3\mathbf{k}$	$\mathbf{F}_2 = -3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$
$A_3$	$\mathbf{r}_3 = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$	$\mathbf{F}_3 = -\mathbf{i} - \mathbf{j} + 2\mathbf{k}$

By introducing forces  $\pm \mathbf{F}_s$ ,  $s = 1, 2, 3$ , at the origin  $O$ , show that the given system can be reduced to a single force  $\mathbf{R} = \sum_{s=1}^3 \mathbf{F}_s$  acting at  $O$  together with a couple of vector moment  $\mathbf{G} = \sum_{s=1}^3 \mathbf{r}_s \times \mathbf{F}_s$ . Find vectors  $\mathbf{R}$  and  $\mathbf{G}$  in terms of  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$ .

Deduce that the system is equivalent to a single resultant force  $\mathbf{R}$  of magnitude  $\sqrt{26}$ .

Show that the lines of action of  $\mathbf{F}_1$  and  $\mathbf{F}_2$  meet at a certain point  $A_0$  with position vector  $\mathbf{r}_0$ , where  $\mathbf{r}_0$  is to be determined. Verify that the line of action of  $\mathbf{F}_3$  also passes through the point  $A_0$ .

Write down the equation of the line of action of the single resultant force  $\mathbf{R}$  in the form  $\mathbf{r} = \mathbf{r}_0 + \gamma \mathbf{R}$ .

Find the position vector of the point where this line meets the  $xy$ -plane.

Hence, show that the Cartesian equations of the line of action of the resultant force  $\mathbf{R}$  may be obtained as  $\frac{x-6}{3} = \frac{y+4}{-4} = \frac{z}{1}$ .

Show further that the Cartesian equation of the plane on which the given system of forces lies is obtained as  $x + 3z = 0$ .

12. A lamina in the shape of a trapezium  $ABCD$  with  $AB$  parallel to  $DC$ ,  $AB = 3a$ ,  $DC = a$  and  $\hat{BAD} = \hat{ABC} = \frac{\pi}{4}$  is immersed vertically in a homogeneous liquid such that  $AB$  is on the free surface of the liquid.

Show that the centre of pressure of the lamina  $ABCD$  is at a distance  $\frac{3a}{5}$  vertically below the mid-point  $E$  of  $AB$ .

A door in the shape of the above lamina  $ABCD$  is made on a vertical side of a tank with  $AB$  horizontal and  $CD$  below  $AB$ . The door is smoothly hinged along  $CD$ . The tank is filled to the level of  $AB$  with a homogeneous liquid of density  $\rho$ .

Find the least force that should be applied at  $E$  in order to keep the door closed with liquid inside the tank.

13. An engine pulls a train along a straight horizontal track against a resistance which at any time is  $k$  times the momentum of the train, where  $k$  is a constant. The engine works at constant power  $9Mkv_0^2$ , where  $M$  is the total mass of the engine and the train. Show that

(i) the maximum speed that the train can attain is  $3v_0$ .

(ii) the time taken for the train to increase the speed from  $v_0$  to  $2v_0$  is  $\frac{1}{2k} \ln\left(\frac{8}{5}\right)$ .

When the train is moving with speed  $U$  its power is cut off and a constant braking force of magnitude  $F$  is applied, in addition to the above resistance. Show that the train will stop at a time  $\frac{1}{k} \ln\left(\frac{F+MkU}{F}\right)$  after the power is cut off.

14. A particle  $P$  of mass  $m$  resting on a smooth horizontal table is connected to a fixed point  $O$  on the table by a light elastic string of natural length  $a$  and modulus of elasticity  $mg$ . When time  $t = 0$ , the particle  $P$  is at a distance  $a$  from  $O$ , with the string just taut, and  $P$  is projected along the table in a direction perpendicular to the initial line of the string, with velocity of magnitude  $U = 2\sqrt{\frac{ga}{3}}$ .

Using the principle of conservation of energy and the principle of conservation of angular momentum about  $O$ , show that

$$\left(\frac{dr}{dt}\right)^2 = U^2 \left(1 - \frac{a^2}{r^2}\right) - \frac{g}{a}(r-a)^2.$$

**Deduce** that

(i) the maximum length of the string is  $2a$ , and the tension in the string at this instant is  $mg$ ,

(ii) the speed of the particle at this instant is  $\frac{U}{2}$ .

Find  $\frac{d^2r}{dt^2}$ , in terms of  $r$  and  $a$  when  $\frac{dr}{dt} \neq 0$ .

More Past Papers at  
[tamilguru.lk](http://tamilguru.lk)

15. Show that

(i) the moment of inertia of a uniform hollow circular cylinder, of mass  $M$  and radius  $a$  about its axis is  $Ma^2$ , and

(ii) the moment of inertia of a uniform circular disc of mass  $m$  and radius  $a$  about the axis through its centre perpendicular to its plane is  $\frac{1}{2}ma^2$ .

A closed container  $C$  made from a thin uniform metal sheet, consists of a right hollow circular cylinder of radius  $a$  and length  $3a$  to which two uniform circular discs, each of radius  $a$ , are attached at the two ends. Show that the radius of gyration,  $k$  of the container  $C$  about its axis is given by  $k^2 = \frac{7}{8}a^2$ .

The container rolls, without slipping, down a rough plane of inclination  $\alpha$  to the horizontal, with its axis horizontal and perpendicular to the lines of greatest slope of the plane.

Show that the acceleration  $f$  of the container  $C$ , in this motion, is given by  $f = \frac{8}{15}g \sin \alpha$ , and that the coefficient of friction  $\mu$  between the container and the plane is such that  $\mu > \frac{8}{15} \tan \alpha$ .

16.(a) Let  $X$  be the number of vehicles leaving a certain car park during a five minutes interval. Suppose  $X$  has the following probability distribution:

$x$	1	2	3	4	5	6
$P(X=x)$	$p$	$2p$	$3p$	$3p$	$2p$	$p$

Find the value of  $p$  and the expected value  $E(X)$  of  $X$ .

Show that the standard deviation of  $X$  is  $\frac{\sqrt{7}}{2}$ .

The random variable  $Y$  is defined by  $Y = 2X + 3$ . Find the expected value  $E(Y)$  of  $Y$  and the standard deviation of  $Y$ .

Also, find the value of  $P(Y \geq E(Y))$ .

(b) The probability that a patient recovers from a delicate surgery is  $\frac{2}{5}$ . Five patients were monitored at random, after they underwent this surgery.

Find the probability that

- (i) at least 3 recover,
- (ii) exactly 2 recover,
- (iii) none recovers.

17.(a) The lifetime,  $T$  hours, of a certain kind of electric lamp can be modelled by the probability density function

$$f(t) = \begin{cases} \frac{1}{a} e^{-\left(\frac{1}{b}\right)t}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

where  $a$  and  $b$  are positive constants.

Show that  $a=b$ .

It is given that 40% of lamps have a lifetime longer than 2000 hours. Find the common value of  $a$  and  $b$ .

Find the distribution function of  $T$ , and hence, show that  $P(T > t+c | T > c) = P(T > t)$ , where  $t \geq 0$  and  $c$  is a positive constant.

(b) The speeds of vehicles passing a certain point  $A$  on a motorway can be taken to be normally distributed. Observations show that of vehicles passing the point  $A$ , 95% are travelling at speed less than  $85 \text{ km h}^{-1}$  and 10% are travelling at less than  $55 \text{ km h}^{-1}$ .

- (i) Find the average speed of the vehicles passing the point  $A$ .
- (ii) Find the percentage of vehicles that travel at a speed more than  $70 \text{ km h}^{-1}$ .

\*\*\*